

第一章 函数与极限

习题 1-1

1. (1) $\left[-\frac{2}{3}, +\infty\right)$; (2) $(-\infty, -1) \cup (-1, 1) \cup (1, +\infty)$;

(3) $[-1, 0) \cup (0, 1]$; (4) $(-2, 2)$;

(5) $[0, +\infty)$; (6) $R \setminus \left\{ \left(k + \frac{1}{2}\right)\pi - 1 \mid k \in Z \right\}$;

(7) $[2, 4]$; (8) $(-\infty, 0) \cup (0, 3]$;

(9) $(-1, +\infty)$; (10) $(-\infty, 0) \cup (0, +\infty)$

2. (1) 不同 (2) 不同 (3) 相同 (4) 不同

3. $\varphi\left(\frac{\pi}{6}\right) = \frac{1}{2}, \varphi\left(\frac{\pi}{4}\right) = \frac{\sqrt{2}}{2}, \varphi\left(-\frac{\pi}{4}\right) = \frac{\sqrt{2}}{2}, \varphi(-2) = 0.$

4. 略

5. (1) 偶函数; (2) 既非奇函数又非偶函数; (3) 偶函数;
(4) 奇函数; (5) 既非奇函数又非偶函数; (6) 偶函数.6. (1) 是周期函数, 周期 $l = 2\pi$; (2) 是周期函数, 周期 $l = \frac{\pi}{2}$;
(3) 是周期函数, 周期 $l = 2$; (4) 不是周期函数;
(5) 是周期函数, 周期 $l = \pi$.

7. (1) $y = x^3 - 1$; (2) $y = \frac{1-x}{1+x}$; (3) $y = \frac{-dx+b}{cx-a}$;
(4) $y = \frac{1}{3} \arcsin \frac{x}{2}$; (5) $y = e^{x-1} - 2$; (6) $y = \log_2 \frac{x}{1-x}$.

8. (1) $y = \sin^2 x, y_1 = \frac{1}{4}, y_2 = \frac{3}{4}$;

(2) $y = \sin 2x, y_1 = \frac{\sqrt{2}}{2}, y_2 = 1$;

(3) $y = \sqrt{1+x^2}, y_1 = \sqrt{2}, y_2 = \sqrt{5}$;

(4) $y = e^{x^2}, y_1 = 1, y_2 = e$;

(5) $y = e^{2x}, y_1 = e^2, y_2 = e^{-2}$.

9. (1) $[-1, 1]$; (2) $\bigcup_{n \in Z} [2n\pi, (2n+1)\pi]$; (3) $[-a, 1-a]$;

(4) 若 $a \in \left(0, \frac{1}{2}\right]$, 则 $D = [a, 1-a]$; 若 $a > \frac{1}{2}$, 则 $D = \emptyset$.

习 题 1-2

- (1) 收敛, 0; (2) 收敛, 0; (3) 收敛, 2; (4) 收敛, 1; (5) 发散;
(6) 收敛, 0; (7) 发散; (8) 发散.
- (1) 必要条件; (2) 一定发散; (3) 不一定收敛, 例如数列 $\{(-1)^n\}$ 有界, 但发散.

习 题 1-3

- (1) 0; (2) -1; (3) 不存在, 因为 $f(0^+) \neq f(0^-)$.
- (1) 错; (2) 对; (3) 错; (4) 错; (5) 对; (6) 对;
- (1) 对; (2) 对; (3) 对; (4) 错; (5) 对; (6) 对; (7) 对; (8) 错.
- $\lim_{x \rightarrow 0^-} f(x) = \lim_{x \rightarrow 0^+} f(x) = 1, \lim_{x \rightarrow 0} f(x) = 1$;
- $\lim_{x \rightarrow 0^-} \varphi(x) = -1, \lim_{x \rightarrow 0^+} \varphi(x) = 1, \lim_{x \rightarrow 0} \varphi(x)$ 不存在.

习 题 1-5

- (1) -9; (2) 0; (3) 0; (4) $\frac{1}{2}$; (5) $2x$; (6) 2; (7) $\frac{1}{2}$;
(8) 0; (9) $\frac{2}{3}$; (10) 2; (11) 2; (12) $\frac{1}{2}$; (13) $\frac{1}{5}$; (14) -1.
- (1) ∞ ; (2) ∞ ; (3) ∞ .
- (1) 0; (2) 0.

习 题 1-6

- (1) ω ; (2) 3; (3) $\frac{2}{5}$; (4) 1; (5) 2; (6) x .
- (1) $\frac{1}{e}$; (2) e^2 ; (3) e^2 ; (4) e^{-k} .

习 题 1-7

- 当 $x \rightarrow 0$ 时, $x^2 - x^3$ 是比 $2x - x^2$ 高阶的无穷小.
- 当 $x \rightarrow 0$ 时, $(1 - \cos x)^2$ 是比 $\sin^2 x$ 高阶的无穷小.
- (1) 同阶, 不等价; (2) 等价无穷小.
- (1) $\frac{3}{2}$; (2) $0(m < n), 1(m = n), \infty(m > n)$; (3) $\frac{1}{2}$; (4) -3.

习 题 1-8

- (1) $f(x)$ 在 $[0, 2]$ 上连续;
(2) $f(x)$ 在 $(-\infty, -1)$ 与 $(-1, +\infty)$ 内连续, $x = -1$ 为跳跃间断点.

2. (1) $x=1$ 为可去间断点, $x=2$ 为第二类间断点;
 (2) $x=0$ 和 $x=k\pi+\frac{\pi}{2}$ 为可去间断点, $x=k\pi(k \neq 0)$ 为第二类间断点;
 (3) $x=0$ 为第二类间断点;
 (4) $x=1$ 为第一类间断点.

习 题 1-9

1. (1) $\sqrt{5}$; (2) 1; (3) 0; (4) $\frac{1}{2}$; (5) 2; (6) $\cos \alpha$; (7) 1; (8) $-\frac{1}{3}$.
 2. (1) 1; (2) 0; (3) \sqrt{e} ; (4) e^3 ; (5) $e^{\frac{3}{2}}$; (6) $\frac{1}{2}$; (7) $\frac{1}{e}$; (8) -6.
 3. $a=1$.

总 习 题 一

1. (1) 必要, 充分; (2) 充分必要.
 2. 1.
 3. (1) (B); (2) (B).
 4. (1) ∞ ; (2) $\frac{1}{2}$; (3) e ; (4) $\frac{1}{2}$; (5) $\sqrt[3]{abc}$; (6) 1; (7) $\frac{1}{a}$; (8) -2.
 5. $a=0$
 6. 略

第二章 导数与微分

习 题 2-1

1. (1) $-f'(x_0)$; (2) $f'(0)$; (3) $2f'(x_0)$.
 2. (B)
 3. (1) $4x^3$; (2) $\frac{2}{3}x^{\frac{1}{3}}$; (3) $1.6x^{0.6}$; (4) $-\frac{1}{2}x^{-\frac{3}{2}}$
 (5) $-\frac{2}{x^3}$; (6) $\frac{16}{5}x^{\frac{11}{5}}$; (7) $\frac{1}{6}x^{-\frac{5}{6}}$
 4. $12m/s$.
 5. 略

$$6. k_1 = y' \Big|_{x=\frac{2}{3}\pi} = -\frac{1}{2}, k_2 = y' \Big|_{x=\pi} = -1.$$

$$7. \text{切线方程为 } \frac{\sqrt{3}}{2}x + y - \frac{1}{2}\left(1 + \frac{\sqrt{3}}{3}\pi\right) = 0;$$

$$\text{法线方程为 } \frac{2\sqrt{3}}{3}x - y + \frac{1}{2} - \frac{2\sqrt{3}}{9}\pi = 0.$$

$$8. x - y + 1 = 0.$$

9. (1) 在 $x=0$ 处连续, 不可导; (2) 在 $x=0$ 处连且可导.

10. $a=2, b=-1$.

11. $f'_+(0)=0, f'_-(0)=-1, f'(0)$ 不存在.

$$12. f'(x) = \begin{cases} \cos x, & x < 0, \\ 1, & x \geq 0. \end{cases}$$

习 题 2-2

1. (1) $3x^2 - \frac{28}{x^5} + \frac{2}{x^2}$; (2) $15x^2 - 2^x \ln 2 + 3e^x$; (3) $\sec x(2 \sec x + \tan x)$;

(4) $\cos 2x$; (5) $x(2 \ln x + 1)$; (6) $3e^x(\cos x - \sin x)$;

(7) $\frac{1 - \ln x}{x^2}$; (8) $\frac{e^x(x-2)}{x^3}$;

(9) $2x \ln x \cos x + x \cos x - x^2 \ln x \sin x$; (10) $\frac{1 + \sin t + \cos t}{(1 + \cos t)^2}$

2. (1) $y'|_{x=\frac{\pi}{6}} = \frac{\sqrt{3}+1}{2}, y'|_{x=\frac{\pi}{4}} = \sqrt{2}$;

(2) $\frac{\sqrt{2}}{4} \left(1 + \frac{\pi}{2}\right)$;

(3) $f'(0) = \frac{3}{25}, f'(2) = \frac{17}{15}$.

3. (1) $v(t) = v_0 - gt$; (2) $t = \frac{v_0}{g}$.

4. 切线方程为 $2x - y = 0$, 法线方程为 $x + 2y = 0$.

5. (1) $8(2x+5)^3$; (2) $3 \sin(4-3x)$; (3) $-6xe^{-3x^2}$; (4) $\frac{2x}{1+x^2}$; (5) $\sin 2x$;

(6) $-\frac{x}{\sqrt{a^2-x^2}}$; (7) $2x \sec^2(x^2)$; (8) $\frac{e^x}{1+e^{2x}}$; (9) $\frac{2 \arcsin x}{\sqrt{1-x^2}}$; (10) $-\tan x$.

6. (1) $-\frac{1}{\sqrt{x-x^2}}$; (2) $\frac{x}{\sqrt{(1-x^2)^3}}$; (3) $-\frac{1}{2}e^{-\frac{x}{2}}(\cos 3x + 6 \sin 3x)$;

(4) $\frac{|x|}{x^2 \sqrt{x^2-1}}$; (5) $-\frac{2}{x(1+\ln x)^2}$; (6) $\frac{2x \cos 2x - \sin 2x}{x^2}$;

$$(7) \frac{1}{2\sqrt{x-x^2}}; \quad (8) \frac{1}{\sqrt{a^2+x^2}}; \quad (9) \sec x; \quad (10) \csc x.$$

$$7. (1) 2xf'(x^2); \quad (2) \sin 2x[f'(\sin^2 x) - f'(\cos^2 x)]$$

习题 2-3

$$1. (1) 4 - \frac{1}{x^2}; \quad (2) 4e^{2x-1}; \quad (3) -2\sin x - x\cos x;$$

$$(4) -2e^{-t} \cos t; \quad (5) -\frac{a^2}{(a^2-x^2)^{3/2}}; \quad (6) -\frac{2(1+x^2)}{(1-x^2)^2};$$

$$(7) 2\sec^2 x \tan x; \quad (8) \frac{6x(2x^3-1)}{(x^3+1)^3}; \quad (9) 2\arctan x + \frac{2x}{1+x^2};$$

$$(10) \frac{e^x(x^2-2x+2)}{x^3}; \quad (11) 2xe^{x^2}(3+2x^2); \quad (12) -\frac{x}{(1+x^2)^{3/2}}.$$

$$2. f'''(2) = 207360.$$

$$3. (1) 2f'(x^2) + 4x^2 f''(x^2); \quad (2) \frac{f''(x)f(x) - [f'(x)]^2}{[f(x)]^2}.$$

习题 2-4

$$1. (1) \frac{y}{y-x}; \quad (2) \frac{ay-x^2}{y^2-ax}; \quad (3) \frac{e^{x+y}-y}{x-e^{x+y}}; \quad (4) -\frac{e^y}{1+xe^y}.$$

$$2. \text{切线方程为 } x+y-\frac{\sqrt{2}}{2}a=0, \text{ 法线方程为 } x-y=0.$$

$$3. (1) -\frac{1}{y^3}; \quad (2) -\frac{b^4}{a^2y^3}; \quad (3) -2\csc^2(x+y)\cot^3(x+y); \quad (4) \frac{e^{2y}(3-y)}{(2-y)^3}$$

$$4. (1) \left(\frac{x}{1+x}\right)^x \left(\ln \frac{x}{1+x} + \frac{1}{1+x}\right);$$

$$(2) \frac{1}{5} \sqrt[5]{\frac{x-5}{x^2+2}} \left[\frac{1}{x-5} - \frac{2x}{5(x^2+2)} \right];$$

$$(3) \frac{\sqrt{x+2}(3-x)^4}{(x+1)^5} \left[\frac{1}{2(x+2)} - \frac{4}{3-x} - \frac{5}{x+1} \right];$$

$$(4) \frac{1}{2} \sqrt{x \sin x \sqrt{1-e^x}} \left[\frac{1}{x} + \cot x - \frac{e^x}{2(1-e^x)} \right].$$

$$5. (1) \frac{3b}{2a} t; \quad (2) \frac{\cos \theta - \theta \sin \theta}{1 - \sin \theta - \theta \cos \theta}.$$

$$6. \sqrt{3} - 2.$$

$$7. (1) \text{切线方程为 } 2\sqrt{2}x + y - 2 = 0, \text{法线方程为 } \sqrt{2}x - 4y - 1 = 0;$$

$$(2) \text{切线方程为 } 4x + 3y - 12a = 0, \text{法线方程为 } 3x - 4y + 6a = 0.$$

$$8. (1) \frac{1}{t^3}; \quad (2) -\frac{b}{a^2 \sin^3 t}; \quad (3) \frac{4}{9} e^{3t}; \quad (4) \frac{1}{f''(t)}.$$

习 题 2-5

当 $\Delta x = 1$ 时, $\Delta y = 18$, $dy = 11$;

1. 当 $\Delta x = 0.1$ 时, $\Delta y = 1.161$, $dy = 1.1$;

当 $\Delta x = 0.01$ 时, $\Delta y = 0.110601$, $dy = 0.11$.

$$2. (1) \left(-\frac{1}{x^2} + \frac{\sqrt{x}}{x} \right) dx; \quad (2) (\sin 2x + 2x \cos 2x) dx;$$

$$(3) (x^2 + 1)^{\frac{3}{2}} dx; \quad (4) \frac{2 \ln(1-x)}{x-1} dx;$$

$$(5) 2x(1+x)e^{2x} dx; \quad (6) e^{-x} [\sin(3-x) - \cos(3-x)] dx;$$

$$(7) dy = \begin{cases} \frac{dx}{\sqrt{1-x^2}}, & -1 < x < 0, \\ -\frac{dx}{\sqrt{1-x^2}}, & 0 < x < 1; \end{cases} \quad (8) 8x \tan x (1+2x^2) \sec^2(1+2x^2) dx;$$

$$(9) -\frac{2x}{1+x^4} dx; \quad (10) A\omega \cos(\omega t + \varphi) dt.$$

$$3. (1) 2x + C; \quad (2) \frac{3}{2} x^2 + C; \quad (3) \sin t + C; \quad (4) -\frac{1}{\omega} \cos \omega x + C;$$

$$(5) \ln(1+x) + C; \quad (6) -\frac{1}{2} e^{-2x} + C; \quad (7); 2\sqrt{x} + C; \quad (8) \frac{1}{3} \tan 3x + C.$$

总 习 题 二

1. (1) 充分, 必要; (2) 充分必要; (3) 充分必要.

2. $n!$.

3. (1) $f'_-(0) = f'_+(0) = f'(0) = 1$;

(2) $f'_-(0) = 1$, $f'_+(0) = 0$, $f'(0)$ 不存在.

4. 在 $x=0$ 处连续, 不可导.

$$5. (1) \frac{dy}{dx} = -\tan \theta, \frac{d^2y}{dx^2} = \frac{1}{3a} \sec^4 \theta \csc \theta;$$

$$(2) \frac{dy}{dx} = \frac{1}{t}, \frac{d^2y}{dx^2} = -\frac{1+t^2}{t^3}.$$

6. 切线方程为 $x+2y-4=0$, 法线方程为 $2x-y-3=0$.

第三章 微分中定理与导数的应用

习 题 3-1

1. 有分别位于区间 $(1,2)$, $(2,3)$ 及 $(3,4)$ 内的三个根.

2. 略

3. 略

4. 略

习 题 3-2

1. (1) 1; (2) 2; (3) 2; (4) $-\frac{3}{5}$; (5) $-\frac{1}{8}$; (6) $\frac{m}{n} a^{m-n}$;

(7) 1; (8) 3; (9) 1; (10) 1; (11) $\frac{1}{2}$; (12) ∞ ;

(13) $-\frac{1}{2}$; (14) e^a ; (15) 1; (16) 1.

2. 略

3. 略

习 题 3-4

1. 单调减少

2. 单调增加

3. (1) 在 $(-\infty, -1]$, $[3, +\infty)$ 内单调增加, 在 $[-1, 3]$ 上单调减少;

(2) 在 $(0, 2]$ 内单调减少, 在 $[2, +\infty)$ 内单调增加;

(3) 在 $(-\infty, 0)$, $(0, \frac{1}{2}]$, $[1, +\infty)$ 内单调减少, 在 $[\frac{1}{2}, 1]$ 上单调增加;

(4) 在 $(-\infty, +\infty)$ 内单调增加;

4. (D).

5. (1) 略 (2) 略

6. (1) 是凸的; (3) 是凹的; (4) 是凹的.

7. (1) 拐点 $(\frac{5}{3}, \frac{20}{27})$, 在 $(-\infty, \frac{5}{3}]$ 内是凸的, 在 $[\frac{5}{3}, +\infty)$ 内是凹的;

(2) 拐点 $(2, \frac{2}{e^2})$, 在 $(-\infty, 2]$ 内是凸的, 在 $[2, +\infty)$ 内是凹的;

(3) 拐点 $(-1, \ln 2), (1, \ln 2)$, 在 $(-\infty, -1], [1, +\infty)$ 内是凸的, 在 $[-1, 1]$ 上是凹的.

$$8. a = -\frac{3}{2}, b = \frac{9}{2}.$$

习 题 3-5

1. (1) 极大值 $f(-1)=17$, 极小值 $f(3)=-47$;

(2) 极小值 $f(0)=0$;

(3) 极大值 $f(\pm 1)=1$, 极小值 $f(0)=0$;

(4) 极大值 $f\left(\frac{3}{4}\right)=\frac{5}{4}$.

2. 略

3. $a = 2, f\left(\frac{\pi}{3}\right) = \sqrt{3}$ 为极大值.

4. (1) 最大值 $f(4)=80$, 最小值 $f(-1)=-5$;

(2) 最大值 $f(3)=11$, 最小值 $f(2)=-14$;

(3) 最大值 $f\left(\frac{3}{4}\right)=1.25$, 最小值 $f(-5)=-5+\sqrt{6}$.

5. 长为 100m, 宽为 5m.

6. $r = \sqrt[3]{\frac{v}{2\pi}}, h = 2\sqrt[3]{\frac{v}{2\pi}}; d : h = 1 : 1$.

7. 底宽为 $\sqrt{\frac{40}{4+\pi}} = 2.367(\text{m})$.

8. 7200 元

9. 60 元

第四章 不定积分

习 题 4-1

$$1. (1) -\frac{1}{x} + C$$

$$(2) \frac{2}{5}x^{\frac{5}{2}} + C$$

$$(3) 2\sqrt{x} + C$$

$$(4) \frac{3}{10}x^{\frac{10}{3}} + C$$

$$(5) -\frac{2}{3}x^{-\frac{3}{2}} + C$$

$$(6) \frac{m}{m+n}x^{\frac{m+n}{m}} + C$$

$$(7) \frac{5}{4}x^4 + C$$

$$(8) \frac{x^3}{3} - \frac{3}{2}x^2 + 2x + C$$

$$(9) \sqrt{\frac{2h}{g}} + C$$

$$(10) \frac{x^5}{5} + \frac{2}{3}x^3 + x + C \quad (11) \frac{x^3}{3} + \frac{2}{5}x^{\frac{5}{2}} - \frac{2}{3}x^{\frac{3}{2}} - x + C \quad (12) 2\sqrt{x} - \frac{4}{3}x^{\frac{3}{2}} + \frac{2}{5}x^{\frac{5}{2}} + C$$

$$(13) 2e^x + 3\ln|x| + C \quad (14) 3\arctan x - 2\arcsin x + C$$

$$(15) e^x - 2\sqrt{x} + C \quad (16) \frac{3^x e^x}{\ln 3 + 1} + C \quad (17) 2x - \frac{5\left(\frac{2}{3}\right)^x}{\ln 2 - \ln 3} + C$$

$$(18) \tan x - \sec x + C \quad (19) \frac{x + \sin x}{2} + C \quad (20) \frac{1}{2}\tan x + C$$

$$(21) \sin x - \cos x + C \quad (22) -(\cot x + \tan x) + C \quad (23) -\cot x - x + C$$

$$(24) -\cos \theta + \theta + C \quad (25) x - \arctan x + C \quad (26) x^3 - x + \arctan x + C$$

习 题 4-2

$$1. (1) \frac{1}{a} \quad (2) \frac{1}{7} \quad (3) \frac{1}{2} \quad (4) \frac{1}{10} \quad (5) -\frac{1}{2} \quad (6) \frac{1}{12} \quad (7) \frac{1}{2} \quad (8) -2$$

$$(9) -\frac{2}{3} \quad (10) \frac{1}{5} \quad (11) -\frac{1}{5} \quad (12) \frac{1}{3} \quad (13) -1 \quad (14) -1$$

$$2. (1) \frac{1}{5}e^{5t} + C \quad (2) -\frac{1}{8}(3-2x)^4 + C \quad (3) -\frac{1}{2}\ln|1-2x| + C$$

$$(4) -\frac{1}{2}(2-3x)^{\frac{2}{3}} + C \quad (5) -\frac{1}{a}\cos ax - be^{\frac{x}{b}} + C \quad (6) -2\cos\sqrt{t} + C$$

$$(7) -\frac{1}{2}e^{-x^2} + C \quad (8) \frac{1}{2}\sin(x^2) + C \quad (9) -\frac{1}{3}(2-3x^2)^{\frac{1}{2}} + C$$

$$(10) -\frac{3}{4}\ln|1-x^4| + C \quad (11) \frac{1}{2}\ln(x^2+2x+5) + C \quad (12) -\frac{1}{3\omega}\cos^3(\omega t + \varphi) + C$$

$$(13) \frac{1}{2\cos^2 x} + C \quad (14) \frac{3}{2}\sqrt[3]{(\sin x - \cos x)^2} + C \quad (15) \frac{1}{11}\tan^{11} x + C$$

$$(16) \ln|\ln \ln x| + C \quad (17) -\frac{1}{\arcsin x} + C \quad (18) -\frac{10^{2\arccos x}}{2\ln 10} + C$$

$$(19) -\ln|\cos\sqrt{1+x^2}| + C \quad (20) (\arctan\sqrt{x})^2 + C \quad (21) -\frac{1}{x\ln x} + C$$

$$(22) \ln|\tan x| + C \quad (23) \frac{1}{2}(\ln \tan x)^2 + C \quad (24) \sin x - \frac{\sin^3 x}{3} + C$$

$$(25) \frac{t}{2} + \frac{1}{4\omega}\sin 2(\omega t + \varphi) + C \quad (26) \frac{1}{2}\cos x - \frac{1}{10}\cos 5x + C$$

(27) $\frac{1}{3}\sin\frac{3x}{2} + \sin\frac{x}{2} + C$

(28) $\frac{1}{4}\sin 2x - \frac{1}{24}\sin 12x + C$

(29) $\frac{1}{3}\sec^3 x - \sec x + C$

(30) $\arctan e^x + C$

(31) $\frac{1}{2}\arcsin\frac{2x}{3} + \frac{1}{4}\sqrt{9-4x^2} + C$

(32) $\frac{x^2}{2} - \frac{9}{2}\ln(x^2+9) + C$

(33) $\frac{1}{2\sqrt{2}}\ln\left|\frac{\sqrt{2x-1}}{\sqrt{2x+1}}\right| + C$

(34) $\frac{1}{3}\ln\left|\frac{x-2}{x+1}\right| + C$

(35) $\frac{2}{3}\ln|x-2| + \frac{1}{3}\ln|x+1| + C$

(36) $\frac{a^2}{2}\left(\arcsin\frac{x}{a} - \frac{x}{a^2}\sqrt{a^2-x^2}\right) + C$

(37) $\arccos\frac{1}{|x|} + C$

(38) $\frac{x}{\sqrt{1+x^2}} + C$

(39) $\sqrt{x^2-9} - 3\arccos\frac{3}{|x|} + C$

(40) $\sqrt{2x} - \ln(1+\sqrt{2x}) + C$

习 题 4-3

1. $-x\cos x + \sin x + C$

2. $x(\ln x - 1) + C$

3. $x\arcsin x + \sqrt{1-x^2} + C$

4. $-e^{-x}(x+1) + c$

5. $\frac{1}{3}x^3\ln x - \frac{1}{9}x^3 + C$

6. $\frac{e^{-x}}{2}(\sin x - \cos x) + C$

7. $-\frac{2}{17}e^{-2x}\left(\cos\frac{x}{2} + 4\sin\frac{x}{2}\right) + C$

8. $2x\sin\frac{x}{2} + 4\cos\frac{x}{2} + C$

9. $\frac{1}{3}x^3\arctan x - \frac{1}{6}x^2 + \frac{1}{6}\ln(1+x^2) + C$

10. $-\frac{1}{2}x^2 + x\tan x + \ln|\cos x| + C$

11. $x^2\sin x + 2x\cos x - 2\sin x + C$

12. $-\frac{e^{-2t}}{2}\left(t + \frac{1}{2}\right) + C$

13. $x\ln^2 x - 2x\ln x + 2x + C$

14. $-\frac{1}{4}x\cos 2x + \frac{1}{8}\sin 2x + C$

15. $\frac{x^3}{6} + \frac{1}{2}x^2\sin x + x\cos x - \sin x + C$

16. $\frac{1}{2}(x^2-1)\ln(x-1) - \frac{1}{4}x^2 - \frac{1}{2}x + C$

17. $-\frac{1}{2}\left(x^2 - \frac{3}{2}\right)\cos 2x + \frac{x}{2}\sin 2x + C$

18. $-\frac{1}{x}(\ln^3 x + 3\ln^2 x + 6\ln x + 6) + C$

19. $3e^{\sqrt[3]{x}}(\sqrt[3]{x^2} - 2\sqrt[3]{x} + 2) + C$

20. $\frac{x}{2}(\cos \ln x + \sin \ln x) + C$

21. $x(\arcsin x)^2 + 2\sqrt{1-x^2} \arcsin x - 2x + C$

22. $\frac{1}{2}e^x - \frac{1}{5}e^x \sin 2x - \frac{1}{10}e^x \cos 2x + C$

23. $\frac{1}{2}x^2\left(\ln^2 x - \ln x + \frac{1}{2}\right) + C$

24. $\frac{2}{3}(\sqrt{3x+9}-1)e^{\sqrt{3x+9}} + C$

第五章 定积分

习题 5-1

1. 略

2. (1) $\frac{1}{2}t^2$ (2) 21 (3) $\frac{5}{2}$ (4) $\frac{9\pi}{2}$

3. (1) 6 (2) -2 (3) -3 (4) 5

4. (1) $6 \leq \int_1^4 (x^2 + 1) dx \leq 51$ (2) $\pi \leq \int_{\frac{\pi}{4}}^{\frac{5\pi}{4}} (1 + \sin^2 x) dx \leq 2\pi$

5. (1) $\int_0^1 x^2 dx$ 较大 (2) $\int_1^2 x^3 dx$ 较大 (3) $\int_1^2 \ln x dx$ 较大

(4) $\int_0^1 x dx$ 较大 (5) $\int_0^1 e^x dx$ 较大

习题 5-2

1. 0, $\frac{\sqrt{2}}{2}$ 2. $\cot t$ 3. $\frac{\cos x}{\sin x - 1}$ 4. 当 $x=0$ 时

5. (1) $2x\sqrt{1+x^4}$ (2) $\frac{3x^2}{\sqrt{1+x^{12}}} - \frac{2x}{\sqrt{1+x^8}}$ (3) $(\sin x - \cos x) \cdot \cos(\pi \sin^2 x)$

6. (C)

7. (1) $a\left(a^2 - \frac{a}{2} + 1\right)$ (2) $\frac{21}{8}$ (3) $\frac{271}{6}$ (4) $\frac{\pi}{6}$ (5) $\frac{\pi}{3}$ (6) $\frac{\pi}{3a}$

(7) $\frac{\pi}{6}$ (8) $\frac{\pi}{4} + 1$ (9) -1 (10) $1 - \frac{\pi}{4}$ (11) 4 (12) $\frac{8}{3}$

8. (1) 1 (2) 2

9. 1

习题 5-3

1. (1) 0 (2) $\frac{51}{512}$ (3) $\frac{1}{4}$ (4) $\pi - \frac{4}{3}$

(5) $\frac{\pi}{6} - \frac{\sqrt{3}}{8}$ (6) $\frac{\pi}{2}$ (7) $\sqrt{2}(\pi + 2)$ (8) $1 - \frac{\pi}{4}$

(9) $\frac{\pi}{16}a^4$ (10) $\sqrt{2}-\frac{2\sqrt{3}}{3}$ (11) $\frac{1}{6}$ (12) $2+2\ln\frac{2}{3}$

(13) $1-2\ln 2$ (14) $(\sqrt{3}-1)a$ (15) $1-e^{\frac{1}{2}}$ (16) $2(\sqrt{3}-1)$

(17) $\frac{\pi}{2}$ (18) $\frac{\pi}{4}+\frac{1}{2}$ (19) 0 (20) $\frac{3}{2}\pi$

(21) $\frac{\pi^3}{324}$ (22) 0 (23) $\frac{2}{3}$ (24) $\frac{4}{3}$

(25) $2\sqrt{2}$ (26) 4

2.(1) $1-\frac{2}{e}$ (2) $\frac{1}{4}(e^2+1)$ (3) $-\frac{2\pi}{\omega^2}$ (4) $\left(\frac{1}{4}-\frac{\sqrt{3}}{9}\right)\pi+\frac{1}{2}\ln\frac{3}{2}$

(5) $4(2\ln 2-1)$ (6) $\frac{\pi}{4}-\frac{1}{2}$ (7) $\frac{1}{5}(e^\pi-2)$ (8) $2-\frac{3}{4\ln 2}$

(9) $\frac{\pi^3}{6}-\frac{\pi}{4}$ (10) $\frac{1}{2}(e\sin 1-e\cos 1+1)$ (11) $2\left(1-\frac{1}{e}\right)$

习 题 5-4

1. (1) $\frac{1}{3}$ (2) 发散 (3) $\frac{1}{a}$ (4) $\frac{\pi}{4}$ (5) π

第六章 定积分的应用

习 题 6-2

1. (1) $\frac{1}{6}$ (2) 1 (3) $\frac{32}{3}$ (4) $\frac{32}{3}$

2. (1) $2\pi+\frac{4}{3}, 6\pi-\frac{4}{3}$ (2) $\frac{3}{2}-\ln 2$ (3) $e+\frac{1}{e}-2$ (4) $b-a$

3. $\frac{9}{4}$

4. (1) πa^2 (2) $\frac{3}{8}\pi a^2$ (3) $18\pi a^2$

5.(1) $\frac{3}{10}\pi$ (2) $\frac{\pi^3}{4}-2\pi$ (3) $160\pi^2$

第七章 微积分方程

习 题 7-1

1. (1) 一阶 (2) 二阶 (3) 三阶 (4) 一阶 (5) 二阶 (6) 一阶

2. (1) 是 (2) 是 (3) 不是 (4) 是

3. (1) $y^2-x^2=25$ (2) $y=xe^{2x}$ (3) $y=-\cos x$

习题 7-2

1. (1) $y = e^{cx}$ (2) $y = \frac{1}{2}x^2 + \frac{1}{5}x^3 + C$
- (3) $\arcsin y = \arcsin x + C$ (4) $\frac{1}{y} = a \ln|x+a-1| + C$
- (5) $\tan x \tan y = C$ (6) $10^{-y} + 10^x = C$
- (7) $(e^x + 1)(e^y - 1) = C$ (8) $\sin x \sin y = C$
- (9) $3x^4 + 4(y+1)^3 = C$ (10) $(x-4)y^4 = Cx$
2. (1) $e^y = \frac{1}{2}(e^{2x} + 1)$ (2) $\cos x - \sqrt{2} \cos y = 0$
- (3) $\ln y = \tan \frac{x}{2}$ (4) $(1+e^x)\sec y = 2\sqrt{2}$
- (5) $x^2y = 4$

习题 7-3

1. (1) $y + \sqrt{y^2 - x^2} = Cx^2 (x > 0), y - \sqrt{y^2 - x^2} = C(x < 0)$
- (2) $\ln \frac{y}{x} = Cx + 1$
- (3) $y^2 = x^2(2\ln|x| + C)$

习题 7-4

1. (1) $e^{-x}(x+c)$ (2) $\frac{x^2}{3} + \frac{3x}{2} + 2 + \frac{c}{x}$ (3) $e^{-\sin x}(x+c)$
- (4) $C \cos x - 2 \cos^2 x$ (5) $\frac{\sin x + c}{x^2 - 1}$ (6) $\frac{2}{3} + Ce^{-3\theta}$
- (7) $2 + Ce^{-x^2}$ (8) $\frac{1}{\ln y} (\frac{1}{2} \ln^2 y + c)$ (9) $(x-2)^3 + c(x-2)$
- (10) $\frac{y^2}{2} + Cy^3$
2. (1) $\frac{x}{\cos x}$ (2) $y = \frac{1}{x}(\pi - 1 - \cos x)$ (3) $y \sin x + 5e^{\cos x} = 1$
- (5) $y = \frac{2}{3}(4 - e^{-3x})$ (5) $y = \frac{x^3}{2}(1 - e^{\frac{1}{x^2}-1})$
3. $y = 2(e^x - x - 1)$

第九章 多元函数微分法及其应用

习题 9-1

$$1. (1) 1 \quad (2) \ln 2 \quad (3) -\frac{1}{4} \quad (4) -2 \quad (5) 2 \quad (6) 0.$$

习题 9-2

$$1. (1) \frac{\partial z}{\partial x} = 3x^2y - y^3, \frac{\partial z}{\partial y} = x^3 - 3xy^2;$$

$$(2) \frac{\partial s}{\partial u} = \frac{1}{v} - \frac{v}{u^2}, \frac{\partial s}{\partial v} = \frac{1}{u} - \frac{u}{v^2};$$

$$(3) \frac{\partial z}{\partial x} = \frac{1}{2x\sqrt{\ln(xy)}}, \frac{\partial z}{\partial y} = \frac{1}{2y\sqrt{\ln(xy)}};$$

$$(4) \frac{\partial z}{\partial x} = y[\cos(xy) - \sin(2xy)], \frac{\partial z}{\partial y} = x[\cos(xy) - \sin(2xy)];$$

$$(5) \frac{\partial z}{\partial x} = \frac{2}{y} \csc \frac{2x}{y}, \frac{\partial z}{\partial y} = -\frac{2x}{y^2} \csc \frac{2x}{y};$$

$$(6) \frac{\partial z}{\partial x} = y^2(1+xy)^{y-1}, \frac{\partial z}{\partial y} = (1+xy)^y \left[\ln(1+xy) + \frac{xy}{1+xy} \right];$$

$$(7) \frac{\partial u}{\partial x} = \frac{y}{z} x^{\frac{y}{z}-1}, \frac{\partial u}{\partial y} = \frac{1}{z} x^{\frac{y}{z}} \cdot \ln x, \frac{\partial u}{\partial z} = -\frac{y}{z^2} x^{\frac{y}{z}} \cdot \ln x;$$

$$(8) \frac{\partial u}{\partial x} = \frac{z(x-y)^{x-1}}{1+(x-y)^{2z}}, \frac{\partial u}{\partial y} = -\frac{z(x-y)^{x-1}}{1+(x-y)^{2z}}, \frac{\partial u}{\partial z} = \frac{(x-y)^z \ln(x-y)}{1+(x-y)^{2z}}.$$

2-3.略

$$4. (1) \frac{\partial^2 z}{\partial x^2} = 12x^2 - 8y^2, \frac{\partial^2 z}{\partial y^2} = 12y^2 - 8x^2, \frac{\partial^2 z}{\partial x \partial y} = -16xy;$$

$$(2) \frac{\partial^2 z}{\partial x^2} = \frac{2xy}{(x^2+y^2)^2}, \frac{\partial^2 z}{\partial y^2} = -\frac{2xy}{(x^2+y^2)^2}, \frac{\partial^2 z}{\partial x \partial y} = \frac{y^2-x^2}{(x^2+y^2)^2};$$

$$(3) \frac{\partial^2 z}{\partial x^2} = y^x \cdot \ln^2 y, \frac{\partial^2 z}{\partial y^2} = x(x-1)y^{x-2}, \frac{\partial^2 z}{\partial x \partial y} = y^{x-1}(1+x \ln y);$$

习题 9-3

$$1. (1) \left(y + \frac{1}{y} \right) dx + x \left(1 - \frac{1}{y^2} \right) dy; \quad (2) -\frac{1}{x} e^{\frac{y}{x}} \left(\frac{y}{x} dx - dy \right);$$

$$(3) -\frac{x}{(x^2+y^2)^{3/2}} (ydx - xdy); \quad (4) yzx^{yz-1} dx + zx^{yz} \cdot \ln x dy + yx^{yz} \cdot \ln x dz.$$

2. $\frac{1}{3}dx + \frac{2}{3}dy$.

3. $\Delta z = -0.119, dz = -0.125$.

4. $0.25e$

5. (A)

习题 9-4

1. $\frac{\partial z}{\partial x} = 4x, \frac{\partial z}{\partial y} = 4y$.

2. $\frac{\partial z}{\partial x} = \frac{2x}{y^2} \ln(3x-2y) + \frac{3x^2}{(3x-2y)y^2}, \frac{\partial z}{\partial y} = -\frac{2x^2}{y^3} \ln(3x-2y) - \frac{2x^2}{(3x-2y)y^2}$.

3. $e^{\sin t - 2t^3} (\cos t - 6t^2)$.

4. $\frac{3(1-4t^2)}{\sqrt{1-(3t-4t^3)^2}}$.

5. $\frac{e^x(1+x)}{1+x^2e^{2x}}$.

6. $e^{ax} \sin x$.

7. 略

8. (1) $\frac{\partial u}{\partial x} = 2xf_1' + ye^{xy}f_2', \frac{\partial u}{\partial y} = -2yf_1' + xe^{xy}f_2'$;

(2) $\frac{\partial u}{\partial x} = \frac{1}{y}f_1', \frac{\partial u}{\partial y} = -\frac{x}{y^2}f_1' + \frac{1}{z}f_2', \frac{\partial u}{\partial z} = -\frac{y}{z^2}f_2'$;

(3) $\frac{\partial u}{\partial x} = f_1' + yf_2' + yzf_3', \frac{\partial u}{\partial y} = xf_2' + xzf_3', \frac{\partial u}{\partial z} = xyf_3'$.

9-10. 略

11. $\frac{\partial^2 z}{\partial x^2} = 2f' + 4x^2f'', \frac{\partial^2 z}{\partial x \partial y} = 4xyf'', \frac{\partial^2 z}{\partial y^2} = 2f' + 4y^2f''$.

12. (1) $\frac{\partial^2 z}{\partial x^2} = y^2f_{11}'', \frac{\partial^2 z}{\partial x \partial y} = f_1' + y(xf_{11}'' + f_{12}''), \frac{\partial^2 z}{\partial y^2} = x^2f_{11}'' + 2xf_{12}'' + f_{22}''$;

(2) $\frac{\partial^2 z}{\partial x^2} = f_{11}'' + \frac{2}{y}f_{12}'' + \frac{1}{y^2}f_{22}'', \frac{\partial^2 z}{\partial x \partial y} = -\frac{x}{y^2} \left(f_{12}'' + \frac{1}{y}f_{22}'' \right) - \frac{1}{y^2}f_2'$,

$$\frac{\partial^2 z}{\partial y^2} = \frac{2x}{y^3} f_2' + \frac{x^2}{y^4} f_{22}'';$$

$$(3) \frac{\partial^2 z}{\partial x^2} = 2yf_2' + y^4 f_{11}'' + 4xy^3 f_{12}'' + 4x^2 y^2 f_{22}'' ,$$

$$\frac{\partial^2 z}{\partial x \partial y} = 2yf_1' + 2xf_2' + 2xy^3 f_{11}'' + 2x^3 y f_{22}'' + 5x^2 y^2 f_{12}'' ,$$

$$\frac{\partial^2 z}{\partial y^2} = 2xf_1' + 4x^2 y^2 f_{11}'' + 4x^3 y f_{12}'' + x^4 f_{22}'' .$$

$$(4) \frac{\partial^2 z}{\partial x^2} = e^{x+y} f_3' - \sin x f_1' + \cos^2 x f_{11}'' + 2e^{x+y} \cos x f_{13}'' + e^{2(x+y)} f_{33}'' ,$$

$$\frac{\partial^2 z}{\partial x \partial y} = e^{x+y} f_3' - \cos x \sin y f_{12}'' + e^{x+y} \cos x f_{13}'' - e^{x+y} \sin y f_{32}'' + e^{2(x+y)} f_{33}'' ,$$

$$\frac{\partial^2 z}{\partial y^2} = e^{x+y} f_3' - \cos y f_2' + \sin^2 y f_{22}'' - 2e^{x+y} \sin y f_{23}'' + e^{2(x+y)} f_{33}'' .$$

习题 9-5

$$1. \frac{y^2 - e^x}{\cos y - 2xy}$$

$$2. \frac{x+y}{x-y}$$

$$3. \frac{\partial z}{\partial x} = \frac{yz - \sqrt{xyz}}{\sqrt{xyz} - xy}, \frac{\partial z}{\partial y} = \frac{xz - 2\sqrt{xyz}}{\sqrt{xyz} - xy} .$$

$$4. \frac{\partial z}{\partial x} = \frac{z}{x+z}, \frac{\partial z}{\partial y} = \frac{z^2}{y(x+z)}$$

5.略

6.略

$$7. \frac{2y^2 z e^z - 2xy^3 z - y^2 z^2 e^x}{(e^x - xy)^3}$$

习题 9-8

$$1. \text{极大值: } f(2, -2) = 8 .$$

$$2. \text{极大值: } f(3, 2) = 36 .$$

3. 极大值: $z\left(\frac{1}{2}, \frac{1}{2}\right) = \frac{1}{4}$.

4. 当矩形的边长分别为 $\frac{2p}{3}$ 及 $\frac{p}{3}$ 时, 绕短边旋转所得圆柱体的体积最大.

5. 当长、宽、高都是 $\frac{2a}{\sqrt{3}}$ 时, 可得最大的体积.

第十章 重积分

习题 10-2

1. (1) $\frac{8}{3}$ (2) $\frac{20}{3}$ (3) 1 (4) $-\frac{3\pi}{2}$

2. (1) $\frac{6}{55}$ (2) $\frac{64}{15}$ (3) $e - e^{-1}$ (4) $\frac{13}{6}$

3. (1) $\int_0^1 dx \int_x^1 f(x, y) dy$ (2) $\int_0^4 dx \int_{\frac{x}{2}}^{\sqrt{x}} f(x, y) dy$

(3) $\int_{-1}^1 dx \int_0^{\sqrt{1-x^2}} f(x, y) dy$ (4) $\int_0^1 dy \int_{2-y}^{1+\sqrt{1-y^2}} f(x, y) dx$

(5) $\int_0^1 dy \int_{e^y}^e f(x, y) dx$ (6) $\int_{-1}^0 dy \int_{-2\arcsin y}^{\pi} f(x, y) dx + \int_0^1 dy \int_{\arcsin y}^{\pi - \arcsin y} f(x, y) dx$

4. (1) $\int_0^{\frac{\pi}{4}} d\theta \int_0^{\sec\theta} f(\rho \cos\theta, \rho \sin\theta) \rho d\rho + \int_{\frac{\pi}{4}}^{\frac{\pi}{2}} d\theta \int_0^{\sec\theta} f(\rho \cos\theta, \rho \sin\theta) \rho d\rho$

(2) $\int_{\frac{\pi}{4}}^{\frac{\pi}{3}} d\theta \int_0^{2\sec\theta} f(\rho) \rho d\rho$

(3) $\int_0^{\frac{\pi}{2}} d\theta \int_{(\cos\theta + \sin\theta)^{-1}}^1 f(\rho \cos\theta, \rho \sin\theta) \rho d\rho$

(4) $\int_0^{\frac{\pi}{4}} d\theta \int_{\sec\theta \tan\theta}^{\sec\theta} f(\rho \cos\theta, \rho \sin\theta) \rho d\rho$

5. (1) $\pi(e^4 - 1)$ (2) $\frac{\pi}{4}(2\ln 2 - 1)$ (3) $\frac{3}{64}\pi^2$

第十一章 曲线积分与曲面积分

习题 11-2

1. (1) $-\frac{56}{15}$ (2) $-\frac{\pi}{2}a^3$ (3) $-\frac{14}{15}$

2. (1) $\frac{34}{3}$ (2) 11 (3) 14 (4) $\frac{32}{3}$

3. (1) $\int_L \frac{P(x, y) + Q(x, y)}{\sqrt{2}} ds$ (2) $\int_L \frac{P(x, y) + 2xQ(x, y)}{\sqrt{1+4x^2}} ds$

$$(3) \int_L \left[\sqrt{2x-x^2} P(x,y) + (1-x)Q(x,y) \right] ds$$

习题 11-3

1. (1) $\frac{1}{30}$ (2) 8
 2. (1) $\frac{5}{2}$ (2) 236 (3) 5
 3. (1) 12 (3) $\frac{\pi^2}{4}$ (4) $\frac{\sin 2}{4} - \frac{7}{6}$

第十二章 无穷级数

习题 12-1

1. (1) $\frac{1+1}{1+1^2} + \frac{1+2}{1+2^2} + \frac{1+3}{1+3^2} + \frac{1+4}{1+4^2} + \frac{1+5}{1+5^2} + \dots$
 (2) $\frac{1}{2} + \frac{1 \cdot 3}{2 \cdot 4} + \frac{1 \cdot 3 \cdot 5}{2 \cdot 4 \cdot 6} + \frac{1 \cdot 3 \cdot 5 \cdot 7}{2 \cdot 4 \cdot 6 \cdot 8} + \frac{1 \cdot 3 \cdot 5 \cdot 7 \cdot 9}{2 \cdot 4 \cdot 6 \cdot 8 \cdot 10} + \dots$
 (3) $\frac{1}{5} - \frac{1}{5^2} + \frac{1}{5^3} - \frac{1}{5^4} + \frac{1}{5^5} - \dots$
 (4) $\frac{1!}{1^1} + \frac{2!}{2^2} + \frac{3!}{3^3} + \frac{4!}{4^4} + \frac{5!}{5^5} + \dots$
2. (1) 收敛 (2) 发散 (3) 发散 (4) 发散 (5) 收敛

习题 12-2

1. (1) 发散 (2) 发散 (3) 收敛 (4) 收敛 (5) $a > 1$ 时收敛, $a \leq 1$ 时发散
 2. (1) 发散 (2) 收敛 (3) 收敛 (4) 收敛
 3. (1) 收敛 (2) 收敛 (3) 发散 (4) 收敛 (5) 发散 (6) 发散
 4. (1) 条件收敛 (2) 绝对收敛 (3) 绝对收敛 (4) 条件收敛 (5) 发散

习题 12-3

1. (1) $(-1,1)$ (2) $(-1,1)$ (3) $(-\infty, +\infty)$ (4) $(-3,3)$
 (5) $\left(-\frac{1}{2}, \frac{1}{2}\right)$ (6) $(-1,1)$ (7) $(-\sqrt{2}, \sqrt{2})$ (8) $(4,6)$

2. (1) $\frac{1}{(1-x)^2} (-1 < x < 1)$ (2) $\frac{1}{4} \ln \frac{1+x}{1-x} + \frac{1}{2} \arctan x - x (-1 < x < 1)$
 (3) $\frac{1}{2} \ln \frac{1+x}{1-x} (-1 < x < 1)$ (4) $\frac{x^2}{(1-x)^2} - x^2 - 2x^3 (-1 < x < 1)$

习题 12-4

1. (1) $\frac{e^x - e^{-x}}{2} = \sum_{n=1}^{\infty} \frac{x^{2n-1}}{(2n-1)!}, (-\infty, +\infty)$

$$(2) \ln(a+x) = \ln a + \sum_{n=1}^{\infty} (-1)^{n-1} \frac{1}{n} \left(\frac{x}{a}\right)^n, (-a, a]$$

$$(3) a^x = \sum_{n=0}^{\infty} \frac{(x \ln a)^n}{n!}, (-\infty, +\infty)$$

$$(4) (1+x)\ln(1+x) = x + \sum_{n=2}^{\infty} \frac{(-1)^n x^n}{n(n-1)}, (-1, 1]$$

$$2. \frac{1}{x} = \frac{1}{3} \sum_{n=1}^{\infty} (-1)^n \frac{(x-3)^n}{3^n}, (0, 6)$$

$$3. \frac{1}{x^2 + 3x + 2} = \sum_{n=0}^{\infty} \left(\frac{1}{2^{n+1}} - \frac{1}{3^{n+1}} \right) (x+4)^n, (-6, -2)$$